Chapter 3

BINARY ARITHMETIC AND TWO's COMPLEMENT ARITHMETIC

Lesson 3

BINARY MULTIPLICATION AND DIVISION

Outline

- **Multiplication**: Unsigned Numbers
- Multiplication: Signed Numbers
- Division

Multiplication of 1-bit $\times$ 1-bit

- Dot operator represents an AND operation. AND operations.
- $1 . 1 = 1$ .....(1)
- $1 . 0 = 0$ .....(2)
- $0 . 1 = 0$ .....(3)
- $0 . 0 = 0$ .....(4)

Multiplication of n-bit $\times$ 1-bit

- Perform AND operation on all n-bits with the multiplying bit
- Multiplication by 1_b gives same number as before

Example 1: 1 0 1 1

$\times$ 1

Using Equations (1) and (2)

Answer is 1 0 1 1

Multiplication of n-bit $\times$ 1-bit

Multiplication by 0_b gives all 0s.

Example 2: 1 0 1 1

$\times$ 0

Using Equations (1) and (2)

Answer is 0 0 0 0

Shift left operation by 1-bit

Example 3: 1 0 1 1

Shift Left

Answer is 0110.

(Place of each bit moves to left and msb is discarded.)

Shift left operation by 2-bit

Example 3: 1 0 1 1

Shift Left Twice

Answer is shift once 0110. Shift again 1100

Multiplication of n-bit × m-bit

Step1: Find P0— Multiple *n*-bit by rightmost 1-bit

Steps 2 and 3: Find P1— Multiple *n*-bit by rightmost but 1-bit and shift left once and add with P0 and find S0

Steps 4 and 5: Find P2 — Multiple *n*-bit by rightmost but 2-bits and shift left 2 times, add by S0. and find S1

Multiplication of n-bit $\times$ m-bit

Continue till all m-bits are multiplied

and sum of partial products P0 + P1 + P_{m-1} is found

Example: multiplicand = 10_d (1010_b) and multiplier = 13_d (1101_b).

Step 1: P0 = x x x x 1 0 1 0

Step 2: P1 = x x x 0 0 0 0 _

Step 3 S0 = x x 0 0 1 0 1 0

Step 4 P2 = x x 1 0 1 0 _ _

Example: multiplicand = 10_d (1010_b) and multiplier = 13_d (1101_b).

Step 5 S1 = x 0 1 1 0 0 1 0

Step 6 P3 = x 1 0 1 0 _ _ _

Step 7 S2 = 1 0 0 0 0 0 1 0

M = 1 0 0 0 0 0 1 0 = Decimal 130_d

Outline

- Multiplication: Unsigned Numbers
- **Multiplication**: Signed Numbers
- Division

Signed Numbers

For multiplication use msb as sign bit and remaining bits for a positive number

Signed Multiplication of n-bit × m-bit

Step1: Find P0— Leave msb and Multiple *n–1* bits by rightmost 1-bit

Steps 2 and 3: Find P1— Multiple *n–1* bits by rightmost but 1-bit and shift left once and add with P0 and find S0

Steps 4 and 5: Find P2 — Multiple *n–1* bits by rightmost but 2-bits and shift left 2 times, add by S0. and find S1

Multiplication of n-bit $\times$ m-bit

Continue till all (m –1) bits are multiplied and sum P0 + P1 + P_{m-2} is found

Now find the sign bit msb of the result

- If both msb = 0s, then msb of product = 0
- If msbs = 1 and 0, then msb of product = 1
- If both msb = 1s, then msb of product = 0

Example: multiplicand = 10_d (1010_b) and multiplier = 13_d (1101_b).

Step 1: P0 = x x x x 1 0 1 0

Step 2: P1 = x x x 0 0 0 0 _

Step 3 S0 = x x 0 0 1 0 1 0

Step 4 P2 = x x 1 0 1 0 _ _

Example: multiplicand = 10_d (1010_b) and multiplier = 13_d (1101_b).

Step 5 S1 = x 0 1 1 0 0 1 0

Step 6 P3 = x 1 0 1 0 _ _ _

Step 7 S2 = 1 0 0 0 0 0 1 0

M = 1 0 0 0 0 0 1 0 = Decimal 130_d

Outline

- Multiplication: Unsigned Numbers
- Multiplication: Signed Numbers
- **Division**

Binary Division

- Binary arithmetic division isby successive subtraction

Division Method

Let dividend by X and divisor be Y. When we divide the unsigned format number (integers non fractional numbers)

Division Method

1. Set the initial quotient = 0000.
2. Check if X < Y, if yes, then Q is unchanged and R = X. Stop the process.
3. If X > Y, increment the quotient. [New Q in first cycle is = 0001, second cycle it will be 0010.]

Division Method

4. Find X – Y using two’s complement arithmetic and get the R.
5. Set X = R.Repeat steps 2 to 5 till X < Y.
6. Now Q is the result for the quotient and R =finally left X is the final remainder

Example X ÷ Y = 0111 ÷ 0011

Here X > Y.

tap 1: Q = 0000

Step 2: X > Y and so go to next step.

Step 3: Q = 0001

Step 4: Find X – Y = 0111 – 0011 = 0111 + 1101 = 0100. R = 0100.

Step 5: X = R = 0100.

Example X ÷ Y = 0111 ÷ 0011

Step 6: Repeat steps 2 to 5. We get Q = 0001 + 1 = 0010 and R = 0001.

Answer is Q = 0010 (decimal 2) and R = 0001 (decimal 1) as expected from division of 7 by 3

Division of Signed numbers

- Divide (*n* –1) bits leaving msbs of X and Y
- Now find the sign bit msb of the quotient
- If both msb = 0s, then msb of quotient = 0
- If msbs = 1 and 0, then msb of quotient = 1
- If both msb = 1s, then msb of quotient = 0

Summary

- **Multiplication is found by 1-bit multiplications, finding partial products, shift left the partial products and find the sum**
- **Division is by repeated subtraction**
- Use signed number in multiplication and division

End of Lesson 2 on

BINARY MULTIPLICATION AND DIVISION

THANK YOU